Trinity College

Semester Two Examination, 2017

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

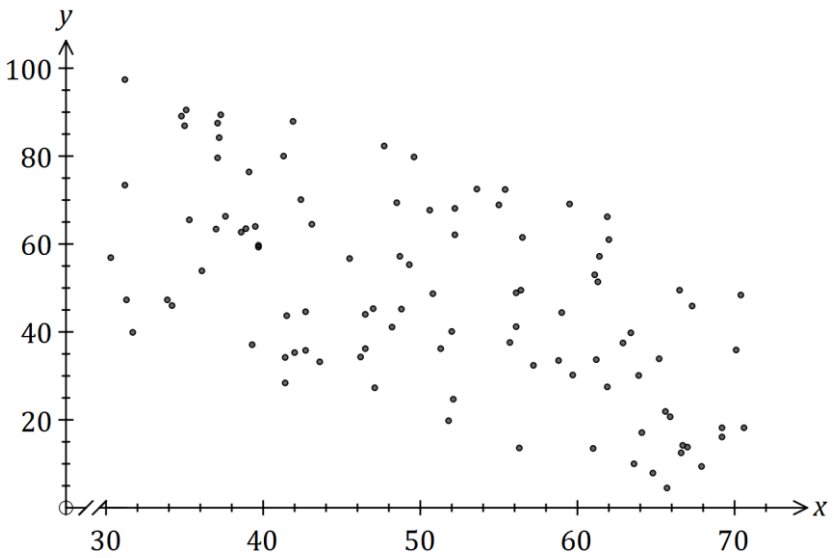
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(5 marks)**

The scatterplot below shows the daily sales of take-away coffees on the x-axis and canned drinks on the y-axis for a market stall.



(a) Describe the association between the variables in terms of strength and direction. (2 marks)

Solution
Association is moderate and negative.
Specific behaviours
✓ indicates strength is moderate ✓ indicates direction is negative

(b) The equation of the least-squares line that fits the data is $y = 108 - 1.2x$. Interpret the slope of this line. (2 marks)

Solution
For every extra coffee sold during a day, the number of canned drinks sold decreases by 1.2.
Specific behaviours
✓ as x increases, y decreases ✓ uses variable names and uses 1.2

(c) Identify a possible non-causal explanation for the association between the variables. (1 mark)

Solution
Coincidence; Confounding due to variable such as temperature, etc.
Specific behaviours
✓ any plausible reason

See next page

Question 9 **(9 marks)**

A researcher sought to determine whether a person's support for a particular political party affected their approval of a proposal to expand the Snowy Hydro scheme.

	Vote Labour	Vote Lib/Nat	Vote other	Total
Approve	156	187	73	416
Disapprove	38	21	22	81
Don't know	80	52	45	177
Total	274	260	140	674

(a) Complete the three missing entries in the table above. (2 marks)

Solution (a)
See table
Specific behaviours
✓ completes third row
✓ completes third column

(b) State which is the response variable and which is the explanatory variable. (2 marks)

Solution
Response: Approval of SH scheme Explanatory: Political party
Specific behaviours
✓ response variable ✓ explanatory variable

(c) Explain why creating a table of column percentages, rather than row percentages, is appropriate in this instance. (1 mark)

Solution
Columns contain explanatory variable
Specific behaviours
✓ uses explanatory variable

(d) Complete the table of column percentages below. (2 marks)

%	Vote Labour	Vote Lib/Nat	Vote other
Approve	57	72	52
Disapprove	14	8	16
Don't know	29	20	32
Total	100	100	100

Solution
See table
Specific behaviours
✓ 2 correct answers
✓ all four correct

(e) Comment, with reasons, on whether any evidence exists to suggest that a person's support for a particular political party affected their approval of the proposal to expand the Snowy Hydro scheme. (2 marks)

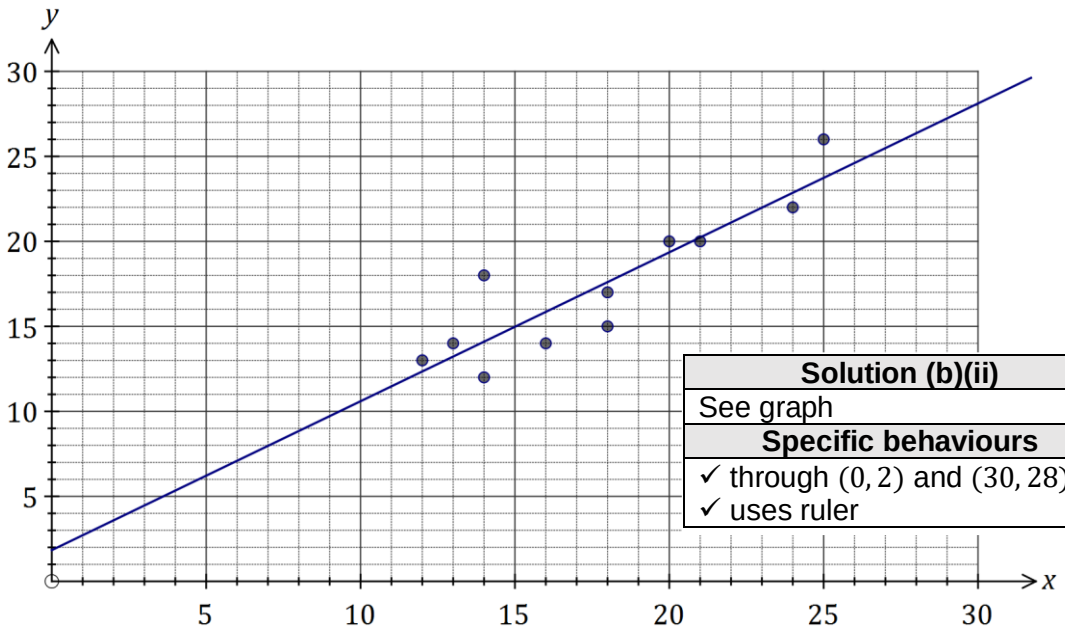
Solution
Yes - evidence of an association exists. There is considerable difference in the column percentages for each response category.
Specific behaviours
✓ states yes ✓ supports answer by noting differences in column percentages

See next page

Question 10 **(8 marks)**

A student was asked to estimate the length of lines generated at random on a computer screen. The actual and estimated lengths, in cm, are shown in the graph and table below.

Actual length, x	18	14	25	20	24	21	18	12	16	14	13
Estimated length, y	17	18	26	20	22	20	15	13	14	12	14



Solution (b)(ii)
See graph
Specific behaviours
✓ through $(0, 2)$ and $(30, 28)$
✓ uses ruler

(a) Calculate the correlation coefficient between the variables x and y. (1 mark)

Solution
$r = 0.894$
Specific behaviours
✓ value to at least 2 dp

(b) A least-squares line can be used to model the relationship between x and y.

(i) Determine the equation of this line. (2 marks)

Solution
$y = 0.876x + 1.84$
Specific behaviours
✓ states equation
✓ both coefficients to at least 2 sf

(ii) Draw this line on the graph. (2 marks)

(c) Predict the student's estimate, to the nearest cm, when the actual length of the line was 8 cm and comment on factors affecting the reliability of this prediction. (3 marks)

Solution
$\hat{y}(8) = 8.84 \sim 9$ cm Prediction is unreliable as despite strong correlation the prediction involves extrapolation beyond the given data.
Specific behaviours
✓ correct value ✓ uses correlation coefficient ✓ uses extrapolation

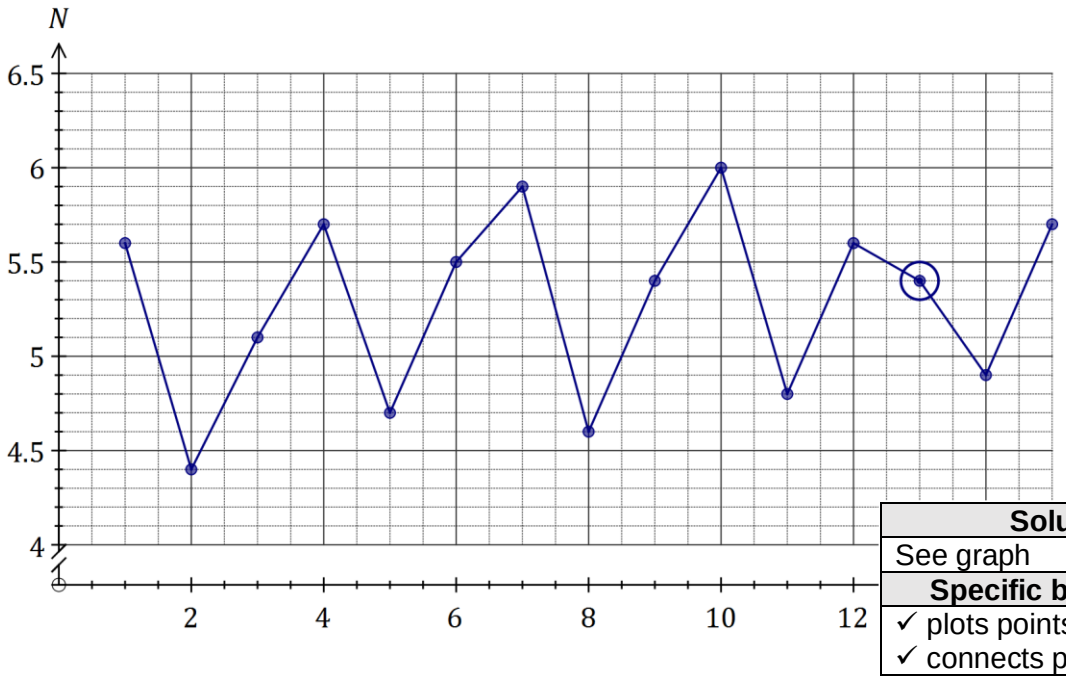
See next page

Question 11 **(10 marks)**

The number of page views, in thousands, of an international website were recorded in the morning (4 am to noon), afternoon (noon to 8 pm) and night (8 pm to 4 am) over several consecutive days.

Day	Period	Time, t	Page views, N, (000's)
Mon	Morning	1	5.6
	Afternoon	2	4.4
	Night	3	5.1
Tue	Morning	4	5.7
	Afternoon	5	4.7
	Night	6	5.5
Wed	Morning	7	5.9
	Afternoon	8	4.6
	Night	9	5.4
Thu	Morning	10	6.0
	Afternoon	11	4.8
	Night	12	5.6
Fri	Morning	13	5.4
	Afternoon	14	4.9
	Night	15	5.7

(a) Use the above data to complete the time series plot on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ plots points
✓ connects points

(b) One of the data points was suspected of being an outlier. Circle this point on the graph and explain why you chose it. (2 marks)

Solution
See graph: $t = 13$ Point does not fit with trend of other morning points.
Specific behaviours
✓ circles point ✓ explanation using morning points

See next page

(c) Describe the trend and seasonality of the time series plot. (2 marks)

Solution
The overall trend of the time series is increasing. Within each day, the page views tend to be highest in the morning, decrease to their lowest in the afternoon and then increase again in the night.
Specific behaviours
✓ describes direction using trend ✓ describes seasonality within daily cycle

(d) Calculate the number of page views representing

(i) the three-point moving average for Tuesday afternoon. (2 marks)

Solution
$\frac{5.7 + 4.7 + 5.5}{3} = 5.3 \text{ thousand}$
Specific behaviours
✓ calculates average ✓ states units

(ii) the six-point centred moving average for Wednesday night. (2 marks)

Solution
$\frac{\frac{5.5}{2} + 5.9 + 4.6 + 5.4 + 6.0 + 4.8 + \frac{5.6}{2}}{6} = 5.375 \text{ thousand}$
Specific behaviours
✓ uses method involving centering ✓ calculates average, with units **only penalise units once in part (d)*

See next page

Question 12 **(12 marks)**

Every afternoon, 20 litres of water is taken from a tank and poured into a small garden pond. At the start of the first day the tank contains 260 L and the pond contains 50 L.

The amount of water in the tank at the start of day n is given by $T_{n+1} = T_n - 20, T_1 = 260$.

(a) Complete the table below. (1 mark)

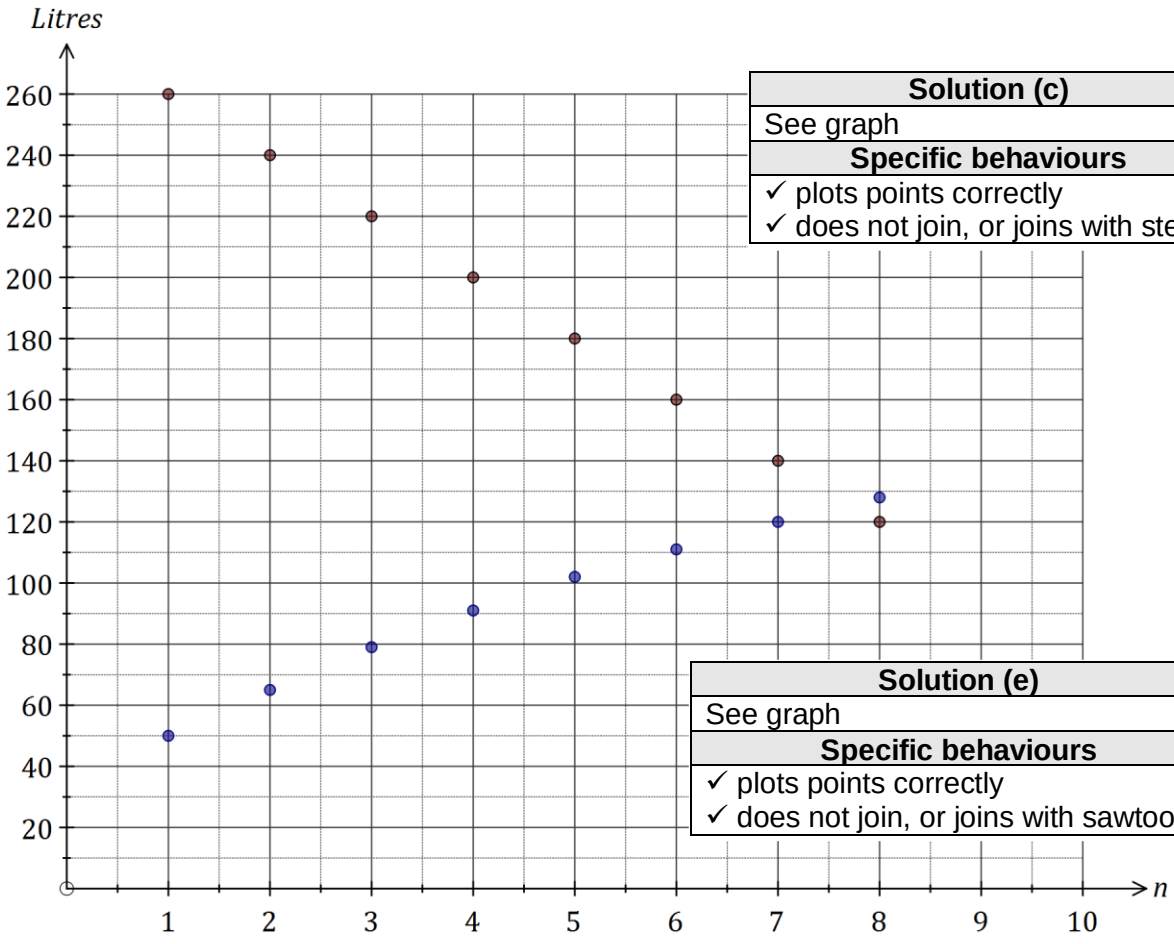
n	1	2	3	4	5	6	7	8
T_n (L)	260	240	220	200	180	160	140	120

Solution (a)
See table
Specific behaviours
✓ all correct

(b) Deduce a rule for the amount of water in the tank at the start of the n^{th} day. (2 marks)

Solution (b)
$T_n = 260 + (n-1)(-20) = 280 - 20n$
Specific behaviours
✓ uses n^{th} term rule ✓ correct rule

(c) Graph the amount of water in the tank at the start of day n on the axes below. (2 marks)



Solution (c)
See graph
Specific behaviours
✓ plots points correctly ✓ does not join, or joins with steps

Solution (e)
See graph
Specific behaviours
✓ plots points correctly ✓ does not join, or joins with sawtooth

See next page

Due to the combined effects of evaporation and water being added from the tank, the amount of water in the pond, in ltres, at the start of day n is given by $P_{n+1} = 0.9P_n + 20, P_1 = 50, n \leq 14$.

(d) Complete the table below, writing all amounts to the nearest litre. (2 marks)

n	1	2	3	4	5	6	7	8
P_n (L)	50	65	79	91	102	111	120	128

Solution
See table
Specific behaviours
✓ correct values for $n = 1, 2$
✓ all correct

(e) Add the amount of water in the pond at the start of day n to the graph in (c). (2 marks)

(f) At the start of which day did the amount of water in the pond first exceed the amount of water in the tank? (1 mark)

Solution
At the start of day 8.
Specific behaviours
✓ correct day

(g) Determine the maximum amount of water in the pond and when this occurs. (2 marks)

Solution
Tank will be empty at start of day 14, at which time pond contains 161.9 L.
Specific behaviours
✓ correct amount
✓ correct day

See next page

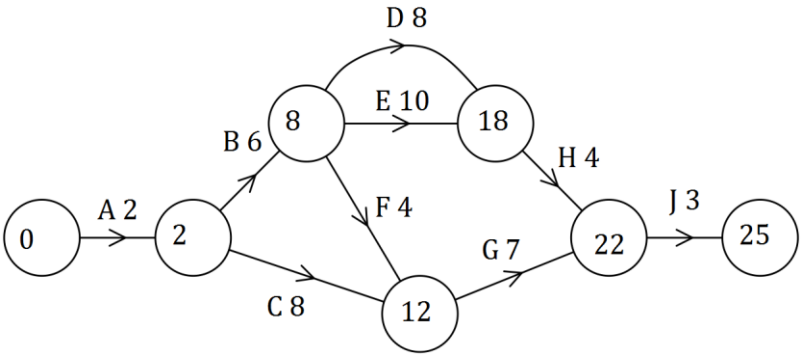
Question 13 **(9 marks)**

The tasks involved in a construction project are shown in the table below.

Task	A	B	C	D	E	F	G	H	J
Duration (days)	2	6	8	8	10	4	7	4	3
Immediate predecessors	-	A	A	B	B	B	F, C	D, E	H, G

(a) Complete the project network below. (3 marks)

Solution
See diagram
Specific behaviours
✓ D, E, F added
✓ all edges, labelled
✓ all correct with direction



(b) List the tasks on the critical path and state the minimum completion time for the project. (2 marks)

Solution
Critical path is A - B - E - H - J MCT is 25 days
Specific behaviours
✓ critical path
✓ MCT

(c) If the project is completed in the minimum possible time, determine

(i) the earliest start time for task G. (1 mark)

Solution
Day 12.
Specific behaviours
✓ correct day

(ii) the latest start time for task F. (1 mark)

Solution
$22 - 7 - 4 = 11$. Day 11.
Specific behaviours
✓ correct day

(iii) the task with the largest float time, and what this float time is. (2 marks)

Solution
Task C - float time of 5 days.
Specific behaviours
✓ task
✓ float time

See next page

Question 14 **(8 marks)**

A student took out a car loan of \$9 500 and made monthly repayments of \$420. The table below shows the progress of the loan for the first few months, with repayments and interest applied at the end of each month.

Month (n)	Balance at start of month (T_n)	Interest	Repayment	Balance carried forward to start of next month
1	9 500.00	104.50	420.00	9 184.50
2	9 184.50	101.03	420.00	8 865.53
3	8 865.53	97.52	420.00	8 543.05
4	8 543.05	$\boldsymbol{A}$	420.00	$\boldsymbol{B}$

(a) Determine the monthly interest rate. (1 mark)

Solution
$104.50 \div 9500 = 0.011$, or 1.1%
Specific behaviours
✓ calculates rate

(b) The recurrence relation to model the loan balance, T_n, in the second column of the table is $T_{n+1} = aT_n - b,\ T_1 = 9\,500$. State the values of a and b. (2 marks)

Solution
$a = 1.011,\ b = 420$
Specific behaviours
✓ states a ✓ states b

(c) Determine the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ in the table. (2 marks)

Solution
$A = 8\,543.05 \times 0.011 = \$93.97$ $B = 8\,543.05 + 93.97 - 420 = \$8\,217.02$
Specific behaviours
✓ calculates A ✓ calculates B

(d) Calculate the balance of the loan at the start of month 6. (2 marks)

Solution
$T_6 = 7887.41$ - balance is \$7887.41
Specific behaviours
✓ uses correct term of sequence ✓ calculates balance

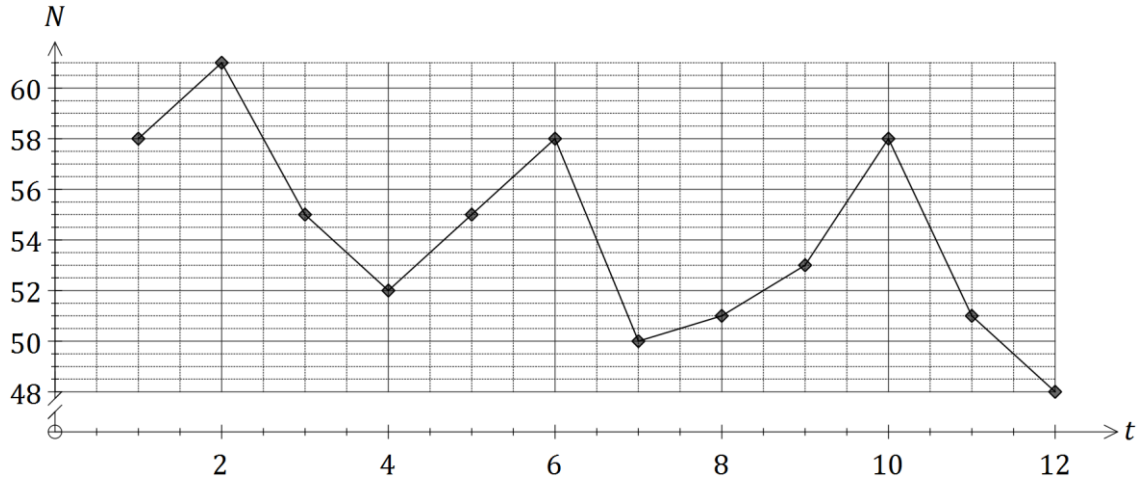
(e) If the student increased the amount of each repayment, comment on how this would change the total interest accumulated over the life of the loan. (1 mark)

Solution
The total amount of interest would decrease.
Specific behaviours
✓ states total interest would decrease

See next page

Question 15 **(9 marks)**

The number of false alarms received by a monitoring station each quarter over the past three years is shown in the graph and table below.



Year	Quarter	Time period (t)	Number of false alarms (N)	Yearly mean	Percent of yearly mean
2014	1	1	58	56.5	$\boldsymbol{A}$
	2	2	61		108.0
	3	3	55		97.3
	4	4	52		92.0
2015	1	5	$\boldsymbol{B}$	53.5	102.8
	2	6	58		108.4
	3	7	50		93.5
	4	8	51		95.3
2016	1	9	53	$\boldsymbol{C}$	101.0
	2	10	58		110.5
	3	11	51		97.1
	4	12	48		91.4

(a) Calculate the values of the entries $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in the table. (3 marks)

Solution
$A = 58 \div 56.5 \times 100 = 102.7$ (or $400 - 108 - 97.3 - 92$) $\frac{B+58+50+51}{4} = 53.5 \Rightarrow B = 55$ $C = (53 + 58 + 51 + 48) \div 4 = 52.5$
Specific behaviours
✓ calculates A ✓ calculates B ✓ calculates C

See next page

(b) Three of the four seasonal indices, calculated using the average percentage method, are shown in the table below. Calculate the missing index for quarter 3 and interpret its value. (2 marks)

Quarter	1	2	3	4
Seasonal index	1.02	1.09	0.96	0.93

Solution
Index for Q3: $4 - 1.02 - 1.09 - 0.93 = 0.96$ This index means that in the third quarter, the number of false alarms is slightly below the yearly average.
Specific behaviours
✓ index calculation ✓ explanation of below average

(c) Calculate the deseasonalised number of false alarms for the fourth quarter of 2015. (1 mark)

Solution
Deseasonalised false alarms: $\frac{51}{0.93} = 54.8$
Specific behaviours
✓ divides by seasonal index

(d) The equation of the least-squares line to forecast the deseasonalised number of false alarms, n, is $n = 57.1 - 0.45t$.

Forecast the **actual** number of false alarms in the second quarter of 2018, assuming that the above seasonality and trends continue. (3 marks)

Solution
Trend: $n = 57.1 - 0.45(18) = 49$ Seasonal adjustment: $49 \times 1.09 = 53.41$ Expect 53 false alarms
Specific behaviours
✓ calculates correct trend value ✓ makes seasonal adjustment ✓ rounds to nearest whole number

See next page

Question 16 **(7 marks)**

A retiree plans to start a pension fund with \$750 000 and then withdraw an annuity of \$55 000 one year later and then at subsequent yearly intervals. The fund is expected to grow at a rate of 5.4% per annum.

(a) Write a linear recurrence relation to model the total amount in the pension fund, T_n, directly after the n^{th} withdrawal. (3 marks)

Solution
$T_{n+1} = T_n \times 1.054 - 55000, \quad T_0 = 750000$
Specific behaviours
✓ uses correct growth rate ✓ uses correct withdrawal ✓ includes T_0

(b) Determine the number of years that the retiree can withdraw \$55 000. (2 marks)

Solution

N	25.3493521
I%	5.4
PV	−750000
PMT	55000
FV	0
P/Y	1
C/Y	1

25 years.

Specific behaviours

✓ indicates $n = 25.35$
✓ states 25 years

(c) Determine how much can be withdrawn each year if the retiree only wants the fund to last for 10 years. (2 marks)

Solution

N	10
I%	5.4
PV	−750000
PMT	99024.11695
FV	0
P/Y	1
C/Y	1

Withdraw \$99 024.12

Specific behaviours

✓ calculates payment
✓ answer uses dollars and cents

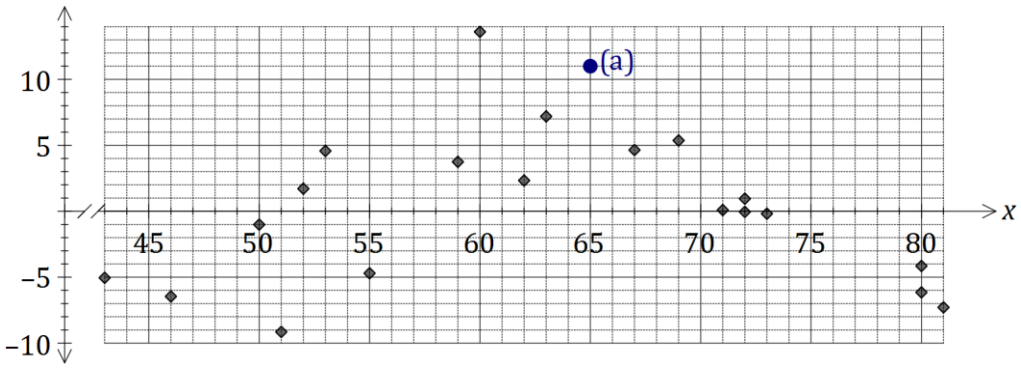
See next page

Question 17 **(9 marks)**

The percentage scores of 20 students in a physics test, x, and a chemistry test, y, were recorded and the following statistics calculated:

- the least-squares line to model the relationship is $\hat{y} = 1.14x - 17.9$
- $r_{xy} = 0.922$

The residual plot for the linear model is shown below.



(a) Another student scored 65% in the physics test and 67% in the chemistry test. Calculate the residual for this student and plot it on the graph above. (3 marks)

Solution
$\hat{y} = 1.14(65) - 17.9 = 56.2$ $67 - 56.2 = 10.8$ See point on graph
Specific behaviours
✓ calculates $\hat{y}$ ✓ calculates residual ✓ plots point

(b) Use the residual plot to explain whether fitting a linear model to the data is appropriate. (2 marks)

Solution
Linear model is not appropriate as a pattern is clearly evident in the residuals.
Specific behaviours
✓ states no ✓ states reason

(c) What percentage of the variation in the chemistry scores can be explained by the variation in the physics scores? (2 marks)

Solution
$0.922^2 \approx 0.85$ so 85%
Specific behaviours
✓ calculates r^2 ✓ correct percentage

(d) Are the variables x and y causally related? Explain your answer. (2 marks)

Solution
No. Although it would be reasonable to expect the observed association, strong association is not by itself sufficient evidence for causation.
Specific behaviours
✓ states no ✓ states reason.

See next page

Question 18 **(6 marks)**

(a) $25 000 is invested in a savings account that earns compound interest of 5.6% per annum. Determine

(i) the total interest added to the investment over five years, if interest is compounded annually. (2 marks)

Solution
$FV = 32\,829.15$ $I = 32\,829.15 - 25000 = \$7\,829.15$
Specific behaviours
✓ calculates future value ✓ calculates interest

(ii) the least time, in months, for the investment to accrue a minimum of $12 000 interest when interest is compounded monthly. (2 marks)

Solution
$t = 84.2$ $t = 85$ months
Specific behaviours
✓ states time as decimal ✓ states required time, with units

(b) $25 000 is invested in a savings account offering 7.8% per annum compounded quarterly.

(i) Calculate the effective interest rate for this account. (1 mark)

Solution
Effective rate is 8.03% pa
Specific behaviours
✓ states effective rate

(ii) Determine the annual rate that a savings account, compounding interest monthly, would need to offer to achieve the same effective rate as in (i). (1 mark)

Solution
Annual rate of 7.75% pa
Specific behaviours
✓ rate that rounds to 7.75%

See next page

Question 19 **(6 marks)**

A lottery winner set up a fund on January 1, 2010, with their prize of $250 000. The balance of the fund grew at an annual rate of 3.5%, compounded monthly. Starting on February 1, 2010, the winner withdrew $500 from the fund on the first day of each month.

(a) Determine the balance in the fund just after the withdrawal was made on January 1, 2011. (2 marks)

Solution

N	12
I%	3.5
PV	−250000
PMT	500
FV	252794.5463
P/Y	12
C/Y	12

Balance was $252 794.55

Specific behaviours

✓ indicates 12 withdrawals
✓ correct balance

(b) Determine the total amount of interest accrued by the fund during 2010. (2 marks)

Solution

PM1	1
PM2	12
I%	3.5
PV	−250000
PMT	500
P/Y	12
C/Y	12
BAL	
INT	
PRN	
ΣINT	8794.546303
ΣPRN	−2794.546303

Total interest was $8 794.55

Specific behaviours

✓ uses change in principal and withdrawals
✓ correct total interest

(c) From January 1, 2011, the annual interest rate applying to the fund decreased and the winner found that the balance of the fund remained the same from month to month. Determine by how much the annual interest rate fell. (2 marks)

Solution

N	12
I%	2.37346889
PV	−252794.55
PMT	500
FV	252794.55
P/Y	12
C/Y	12

$3.5 - 2.373 = 1.127\%$ decrease.

Specific behaviours

✓ calculates new rate
✓ determines change in rate

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____